Dynamical Interactions Among Extrasolar Planets and their Observability in Radial Velocity Data Sets

Report of Working Group IV

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Abstract. For certain multiple-planet systems such as GJ 876 and 55 Cancri, which have been observed for a large number of orbital periods, and which have strong planet-planet gravitational interactions, the approximation that the planets are orbiting on independent Keplerian ellipses is inadequate. This phenomena is of immediate importance to the interpretation of the increasing number of known multiple-planet systems, and it can potentially be used to remove the sin(i) degeneracy. Multiple-planet fitting sparked a great deal of discussion during the working group session on planet-planet interactions. In this report, we provide a self-contained summary of the techniques and the inherent potential of self-consistent dynamical fits to interacting planetary systems.

1. Introduction

The steady accumulation of Doppler radial velocity (RV) observations of nearby stars has made it clear that many systems have more than one planet. The first extrasolar multiple-planet system was discovered around υ Andromedae (Butler et al., 1999), and in the past two years, pairs or triplets of planets have been detected around a number of additional stars, including GJ876 (Marcy et al., 2001), 47 UMa (Fischer et al., 2002), and 55 Cancri (Marcy et al., 2002). Within the current planet catalog, there are 11 multiple-planet systems containing 26 planets (Schneider, 2002). Additional planetary companions may ultimately be found around more than half of the stars with one detected planet (Fischer et al., 2001).

The presence of two or more interacting planets in a system dramatically increases our potential ability to constrain and understand the processes of planetary formation and evolution. Planet-planet interactions can be reasonably subdivided into three categories: (i) interactions during the planet formation and nebular phases, (ii) ongoing secular or resonant interactions which manifest themselves on observable timescales of decades or less, and (iii) long-term dynamical interactions that can sculpt a planetary system over timescales comparable to the lifetime of the parent star. Short-term dynamical interactions (category ii) are of particular interest because they have immediately observable consequences, and indeed, several multiple-planet systems are showing clear signs of ongoing interaction. In this working group report, we summarize the current situation regarding planet-planet interactions that are directly observable within RV data sets.
2. Non-Keplerian Interactions

When confronted with a radial velocity (RV) data set that contains the signature of more than one planet, one is faced with a rather tricky question: How are the individual motions of the planets to be separately pulled out of the composite curve? Isaac Newton, in his analysis of observations of our own solar system, was the first to address the question of dynamical interactions among multiple planets. In Book I, Section II, proposition 69, of the Principia (Newton, 1687), he writes:

“And hence, if several lesser bodies revolve about a greatest one, it can be found that the orbits described will approach closer to elliptical orbits, and the description of the areas will become more uniform [...] if the focus of each orbit is located in the common center of gravity of all the inner bodies.”

In modern terms, one says that to first order, orbits in a multiple-planet system are Keplerian when written in terms of Jacobi coordinates.

One can visually evaluate the validity of the Keplerian approximation in Jacobi coordinates by integrating the full equations of motion for the planets in a given multiple-planet system. The results of four such integrations (for Upsilon Andromedae, HD 168443, 55 Cancri, and GJ 876) are shown in the separate panels of Figure 1. In each case, the integration is started at the epoch of the first published RV observation, and the trajectories of the planets are plotted from that moment through to the present (November 01, 2002). A glance at Figure 1 makes it clear that in the Upsilon Andromedae system (and to a lesser extent in the HD 168443 system) the planetary orbits are dynamically separated to the extent that direct planet-planet perturbations are not very apparent over the timescale of the RV observations. In these cases, as with most of the other known multiple-planet systems, Newton’s approximation that the planets trace fixed elliptical orbits about the center of mass of all interior bodies works very well. For systems such as GJ 876, and 55 Cancri, however, the direct perturbations between the planets give the motions a distinctly non-Keplerian flavor. The planetary orbits are aperiodic, and over the time covered by the RV campaigns, their excursions explore thick elliptical annuli surrounding the star. These annuli arise largely from precession of the orbits. Therefore, in order to determine whether a self-consistent N-body fit to the motion is required, one can simply integrate the nominal initial orbital parameters of the system, and look to see whether the resulting motion consists of nearly fixed ellipses.

3. Multiple Planet Fitting

When planets display significant non-Keplerian motion, self-consistent fitting is necessary in order to characterize the system. One must specify the initial masses, positions, and velocities of the planets relative to the star at the epoch of the first
observation, and then examine how the integrated stellar reflex motion, $v_{\text{model}}$, compares with the observed RV, $v_{\text{obs}}$, at each epoch for which an RV observation exists. Every set of initial conditions for the planets thus corresponds to a particular value of

$$
\chi^2 = \frac{1}{N - M} \sum_{i=1}^{N} \left( \frac{v_{\text{obs}}(t_i) - v_{\text{model}}(t_i)}{\sigma_i} \right)^2
$$

where $N$ is the number of RV observations, $M$ is the number of parameters (osculating orbital elements), and $\sigma_i$ are the individual RV errors for the $N$ observations. The minimum value for $\chi^2$ corresponds to the best system model.

The basic idea is then to vary the initial conditions for the planets so that a minimum value for $\chi^2$ is obtained. A value $\chi^2 \sim 1$ indicates that an adequate model for the system has been found. Because a particular initial condition leads to deterministic motion, a given orbital fit also constitutes a prediction of where future RV measurements will fall. We note that due to a historical quirk, orbital fits to RV data are usually reported in terms of the statistic $\sqrt{\chi^2}$, rather than in terms of $\chi^2$.  

*Figure 1. Orbital motion in the center-of-mass frame for four known multiple-planet systems. In each diagram, the orbital motion has been integrated over the actual time frame of the RV observations.*
3.1. GJ 876: A SUCCESSFUL TEST CASE

To date, the GJ 876 system has provided by far the best and most straightforward opportunity for self-consistent fitting. As reported by Marcy et al. (2001), two-Keplerian fits to the RV data from the Keck telescope and to a combined data set from Keck and Lick, suggest that the star (a $0.32 \, M_\odot$ M dwarf) is accompanied by two planets on $\sim 30$ day and $\sim 60$ day orbits. The RV amplitude variations suggest a minimum mass of order $0.6 \, M_{\text{JUP}}$ for the inner planet, and $1.9 \, M_{\text{JUP}}$ for the outer planet. This combination of a low-order (2:1) commensurability, relatively large planetary masses in comparison to the $0.32 \, M_\odot$ primary, and observations spanning more than thirty orbits of the outer companion, combine to make self-consistent fits necessary to accurately model the system. To see this, consider Figure 2, which compares synthetic RV curves arising from orbital elements given in Marcy et al. (2001). The red line shows the RV curve that results from the super-position of the two Keplerian reflex motions. The black line shows the RV curve resulting from the full three-body integration. After three orbits of the outer planet, the motion begins to deviate noticeably from the dual-Keplerian approximation. After several years, the motions have diverged completely!

As reported in Laughlin and Chambers (2001), and as found independently by Rivera and Lissauer (2001), when planet-planet interactions are strong, one can obtain self-consistent orbital fits through the use of a calculus-based Levenberg-Marquardt minimization algorithm (e.g., Press et al., 1992) driving $N$-body integrations of prospective planetary configurations. If the Levenberg-Marquardt algorithm is started with osculating planetary orbital elements corresponding to the best
dual-Keplerian fit to the radial velocities, then a significant improvement to the fit is rapidly obtained. Our self-consistent model for the combined Keck and Lick RV data from Marcy et al. (2001) has a $\sqrt{\chi^2}$ value of 1.46 and an rms scatter of 13.95 m s$^{-1}$. This fit is shown in Figure 3, and represents a significant improvement to the best dual-Keplerian fit which has $\sqrt{\chi^2} = 1.88$.

Fits which take planet-planet interactions into account can conceivably remove the sin($i$) degeneracy and reveal the true masses of the planets. In the case of GJ 876, the combined RV data favors a co-planar inclination of sin($i$) = 0.775 for the system. There is, however, a broad minimum around this best-fit value. At sin($i$) = 0.55, the $\sqrt{\chi^2} = 1.47$, and the rms scatter is 14.69 ms$^{-1}$. Only for sin($i$) < 0.50 does the fit begin to worsen significantly. This result is in substantial agreement with the result found by Rivera and Lissauer (2001). Future and ongoing RV measurements will allow the system model to be gradually refined.

Nauenberg (2002) performed a similar analysis using the Keck data alone, and argues that the Keck RV subset is best fit by a co-planar system having sin($i$) $\sim$ 1. His rms scatter and $\chi^2$ values are higher than those found by Laughlin and Chambers (2001) and Rivera and Lissauer (2001), however.

It is interesting to remark that if the GJ876 planets are indeed co-planar, and if the orbits are viewed edge-on, then planet-planet interactions will strongly affect the predicted transit epochs. For example, if one uses the best dual-Kepler fit, one finds epochs of JD=2452599.27, JD=2452629.39, and JD=2452659.51 for the next three transits of the inner planet. Using a co-planar $i = 90^\circ$ self-consistent fit to exactly the same RV data, one obtains completely different predictions for the next three transit times; JD-2452607.85, JD=2452638.11, and JD=2452668.28. Further-

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Figure 3. Synthetic RV variations for the best fit generated by a Levenberg-Marquardt scheme applied to the GJ 876 Keck+Lick radial velocities reported by Marcy et al. (2001).
more, the individual transit epochs for GJ 876c (the less massive inner planet) are not equally spaced. Libration generated by the resonant interaction causes delays and early arrivals of the transit center by up to 4 hours on either side of the mean period. A 4-hour delay or advance is highly significant when compared to observation intervals spaced by several minutes. Even more remarkably, if the planets have a mutual inclination relative to one another, it is possible that the node of one of the planets can precess into an inclination where transits begin to occur. This possibility is examined in an upcoming paper.

4. Genetic Algorithms as Applied to Multiple-Planet Fitting

Self-consistent fits to the GJ 876 data set show that the GJ 876 system lies deep within the 2:1 mean motion and secular resonances, with libration angles as small as 5°. Indeed, Lee (2002) have shown that these highly damped librations require that strong dissipative processes were present as the planetary orbits evolved into their present configuration.

It is precisely because the GJ 876 system lies so deeply within the 2:1 resonance that a dual-Keplerian fit provides an initial guess adequate for Levenberg-Marquardt minimization to successfully refine the model of the system. For other planetary configurations, however, especially those which lie near the separatrix surrounding a resonance, small variations in the osculating orbital elements of the planets at the initial epoch lead to very strong sensitivity of $\sqrt{\chi^2}$, coupled with an overall $\sqrt{\chi^2}$ landscape that is topologically rugged on larger scales.

Multiple-planet fitting techniques for RV data sets can be tested by computing a stellar reflex motion arising from a specified configuration of planets, and then sampling this motion at cadences and with uncertainties appropriate to the current RV data sets. For instance, as a representative example, one can adopt the updated Keck Telescope RV data set for GJ 876 (G. W. Marcy 2001, private communication) which includes 63 RV measurements spanning 1532 days. Each individual velocity has an associated measurement uncertainty that is estimated from the in-
Figure 4. Upper panel: Power spectrum corresponding to a synthetic radial velocity data set produced using the hypothetical planetary system shown in Table I, sampled at the epochs and velocity precision of the GJ 876 Keck radial velocities. Lower Panel: Power spectrum corresponding to a single planet on a circular orbit with $K = 75 \text{m/s}$, and $P = 30 \text{d}$, also sampled at the epochs and velocity precision of the GJ 876 data set.

ternal cross-correlation of the lines within the spectra. These uncertainties range from 2.8 to 8.3 m s$^{-1}$, with an rms value of 5.14 m s$^{-1}$.

In Table I, we specify osculating orbital elements of a hypothetical 1:1 resonant pair of planets orbiting a solar mass star. This synthetic system exhibits large (tadpole-like) librations about the equilateral co-orbital resonant configuration, and is thus not well-suited to either a Keplerian fit or to the Levenberg-Marquardt minimization procedure that worked for GJ 876.

Using these orbital elements at the epoch of the first radial velocity point as initial conditions, we integrated the hypothetical system forward in time for the duration spanned by the actual RV observations of GJ 876. We then sampled the RV of the parent star in response to the hypothetical 1:1 resonant pair at the 63 observational epochs. For each point, we then added RV noise drawn from a Gaussian distribution of half-width given by the actual quoted velocity errors. This yields a synthetic RV data set from which we can attempt to reconstruct the orbital parameters of the planets.

The top panel of Figure 4 shows the power spectrum of the synthetic RV data set. There is a single significant peak at the fundamental 30 day period, and the spectrum gives little indication of the presence of two planets in the data. In particular, the accordion-like modulation of the RV curve due to the 300 day libration frequency between the planets is not immediately apparent in the power spectrum, as can be seen by comparing with the power spectrum produced by a single planet.
of \( K = 75 \text{ m s}^{-1} \) on a circular 30 day orbit (bottom panel of Figure 4). In both cases, the power spectrum shows only a single significant peak.

In order to fit the synthetic 1:1 resonant system, we have employed a two-stage method. In the first stage, we use a genetic algorithm (Goldberg, 1989) as implemented in FORTRAN by Carroll (1999)\(^1\) for public domain use. The genetic algorithm starts with an aggregate of osculating orbital elements, each referenced to the epoch of the first RV observation (\( T_0 = 2450602.0931 \)). Each set of elements (genomes) describes a unique three-body integration and an associated radial velocity curve for the central star. The fitness of a particular genome is measured by the \( \chi^2 \) value of its fit to the RV data set. At each generation, the genetic algorithm evaluates the \( \chi^2 \) fit resulting from each parameter set, and cross breeds the best members of the population to produce a new generation.

Because the fundamental 30-day period of the system is clearly visible in the power spectrum, the genetic algorithm is constrained to search for 1:1 resonant configurations in which the initial period ranges for the planets are \( 29 \text{ d} < P_1 < 31 \text{ d} \), and \( 29 \text{ d} < P_2 < 31 \text{ d} \). The initial arguments of periapse and mean anomalies of the two planets are allowed to vary within the allowed \( 2\pi \) range. The RV half-amplitudes of the planets are required to fall in the range \( 0 \text{ m s}^{-1} < K_1 < 150 \text{ m s}^{-1} \) and \( 0 \text{ m s}^{-1} < K_2 < 150 \text{ m s}^{-1} \). The planetary eccentricities are allowed to vary within the full \( 0 < e < 1 \) range.

With these very liberal constraints on the space of osculating initial orbital elements, the Genetic Algorithm rapidly identifies a set of parameters having \( \chi = 1.27 \). This tentative fit is then further improved by use of Levenberg-Marquardt minimization to produce a fit to the data having \( \chi = 0.97 \). This fit is shown in Figure 5, and in the third and fourth columns of Table I. The excellent agreement between the input system and the fitted solution shows that a resonant pair of this type is readily identifiable if it exists within a RV data set comparable to the GJ 876 velocities. As the RV surveys continue, an increasing number of target stars will have velocity data sets of this quality.

### 5. 55 Cancri – A Challenge For Multiple-Planet Fitting

The \( P = 14.65 \text{ d} \), \( M \sin(i) = 0.84 M_{\text{JUP}} \) object orbiting the nearby super-metal-rich \( 0.95 M_\odot \) star 55 Cnc was the fourth RV planet to be discovered (Butler et al., 1997). Soon after the discovery, residuals in the radial velocities indicated the likely presence of additional planet(s) in the system. After five further years of monitoring, the residuals can be modeled as arising from two additional planets, one with a period \( P = 44.3 \text{ d} \) and \( M \sin(i) = 0.21 M_{\text{JUP}} \), and a second with period \( P \sim 5400 \text{ d} \) and \( M \sin(i) = 4.00 M_{\text{JUP}} \) (Marcy et al., 2002). The period ratio of the inner two planets in the Marcy et al. (2002) model is \( P_1 / P_2 = 3.02 \), which is close

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\(^1\) See [http://cuaerospace.com/carroll/ga.html](http://cuaerospace.com/carroll/ga.html)
Figure 5. Synthetic RV variations for the planetary system shown in Table I (solid line). The circular points with small vertical lines corresponding to error bars represent a sample of the system with the properties of the GJ876 data set. A superimposed dotted line line shows a fit to the sampled data obtained with a combined Genetic Algorithm and Levenberg-Marquardt procedure. It is hard to distinguish the difference between these two curves at the resolution of the plot.

to the 3:1 commensurability. This means that over the $\sim 5000$ day timespan that the star has been observed, the planet-planet interactions between the inner and middle planets will tend to add in a constructive way. This is illustrated in Figure 6, which shows the running difference between the RV curve of the star under the influence of summed Kepler motions, and under full four-body motion. The discrepancy between the two versions of the stellar motion grows to $\Delta V > 100 \text{ m s}^{-1}$ after 10 years of observation, which indicates that self-consistent fitting should be required in order to correctly describe a system in this configuration.

Inspired by the successful attempts to fit the GJ 876 RV data set, we first used the Levenberg-Marquardt minimization routine driving a four-body integrator to produce a self-consistent fit. As with GJ 876, the best summed Keplerian fit was used as an initial guess.

One soon finds, however, that the 3-Keplerian fit to the 55 Cancri data does not provide a similarly propitious point of departure for improvement of the orbital fit. When the summed Keplerian fit is used as an initial guess, the code converges to a self-consistent solution with $\chi^2 = 1.85$. This fit is shown in the first column of Table II. Experimentation with the starting conditions shows that there is a very strong sensitivity of $\chi^2$ to small variations in the initial conditions, coupled with a $\chi^2$ landscape that is topologically rugged on large scales. One must therefore resort to alternate methods to locate the true configuration of the system.
We first used a scheme which turns on the planet-planet perturbations in a gradual way, and which was successfully adopted for GJ876 by Rivera and Lissauer (2001). We decreased both the masses of the planets and the magnitudes of the stellar reflex velocities by a factor of $10^6$. This allows the Levenberg-Marquardt N-body code to recover the 3-Keplerian fit listed in the first column of Table II. We then gradually increased both the masses of the planets and the radial velocities in a series of discrete increments. After each increment, we allowed the Levenberg-Marquardt minimization to converge to a self-consistent fit. When the radial velocities have grown to their full observed values, the code produces a self-consistent $\sqrt{\chi^2} = 1.82$ fit, which we list in the second column of Table II. (All of the fits in Table II correspond to Epoch JD 2447578.730, the time at which the first RV observation of the star was made).

With the exception of the eccentricity of the middle planet, which has dropped from its large value of $e = 0.34$, the osculating orbital elements for the self-consistent fits are quite similar to the summed Kepler fit. In contrast to the situation with GJ 876, the imposition of planet-planet interactions has not improved the $\chi^2$ statistic. Furthermore, examination of the 3:1 resonance arguments, $\theta_1 = 3\lambda_c - \lambda_b - 2\sigma_b$, $\theta_2 = 3\lambda_c - \lambda_b - \sigma_c - \sigma_b$, and $\theta_3 = 3\lambda_c - \lambda_b - 2\sigma_c$, show that while the model systems are close to resonance, none of the resonance arguments are librating for any of the fits. This is illustrated in Figure 7, where the time variation of the resonant argument $\theta_1$ is plotted for both the 3-Keplerian fit and for the self-
consistent fits. It seems clear that the 55 Cancri system will likely turn out to be
very interesting dynamically. Even if it is not in resonance today, it likely was in
the resonance in the past, and the fact that it is currently not indicates an intriguing
history, possibly including tidal dissipation.

The failure of the Levenberg-Marquardt routine to significantly improve the $\chi^2$
statistic suggests that a true global minimum in the three-planet parameter space
was simply not located by the locally convergent Levenberg-Marquardt algorithm,
suggesting the use of the globally convergent genetic algorithm, as described in
the previous section. Unfortunately, however Extensive use of the genetic algo-
{}rithm also fails to find a significantly improved $\chi^2$ value over that provided by the
summed Keplerian fit. This is in contrast to the algorithm’s excellent performance
on the above-described test problem. The best fit which we evolved is listed in the
third column of Table II. This fit has $\sqrt{\chi^2}=1.82$, and has osculating orbital elements
which are very similar to the fit obtained by slowly increasing the strength of the
planet-planet interactions (second column of Table II).

In summary, we have investigated three numerical strategies for producing self-
consistent three-planet fits to the 55 Cancri RV data set. All three methods provide

Table II. Three-Planet Models for the 55 Cancri System

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Figure 7. Time behavior of the 3:1 resonant argument $\theta_1 = 3\lambda_x - \lambda_b - 2\pi p$, for Top Panel summed Keplerian fit, Second Panel self-consistent 4-body fit #1 (see Table II), Third Panel self-consistent 4-body fit #2, Bottom Panel self-consistent 4-body fit #3.

fits with $\chi^2$ statistics that are essentially equivalent of the best 3-Kepler fit, and all three fits are in quite good agreement, suggesting that the system lies just outside of the 3:1 resonance. We stress, however, that it is not yet completely clear whether the system is indeed in the 3:1 resonance. Further dynamical fitting and further observation of the star will be required to definitively identify the dynamical relationship between the inner and the middle planet.

6. Conclusions

The most important long-term benefit of self-consistent dynamical fitting techniques for multiple-planet systems lies in their ability to break the $\sin(i)$ degeneracy and thus determine the true masses and mutual inclinations of extrasolar planets. In a few systems, the true masses can be found when a planet transits the parent star, but these cases will be relatively rare. For stars brighter than V=10, for which 3-5 m/s RV precision can be readily obtained, we estimate that there are $\approx 10$ transiting planets with periods less than a week, and $\approx 10$ transiting
planets with periods in the range $7 \text{d} < P < 200 \text{d}$. Self-consistent fitting, on the other hand, can be applied to any system containing more than one planet, given a sufficient baseline of observation. Self-consistent fitting may also help with uncovering the presence of additional planets in a system, or demonstrating that an additional planet (beyond the first two, say) is unnecessary to get an adequate fit.

References


