Multi-Spacecraft Methods of Wave Field Characterisation

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5.1 Introduction

Space plasmas are collisionless and thus waves play a major role in the collective interaction between particles. Also, space plasmas are subject to a variety of instabilities, generating a plethora of different plasma wave modes. Thus, for many physical processes the role of waves and turbulence is likely to be predominant. The simultaneous four-point measurements available from the Cluster mission enable spatio-temporal effects in data sets to be resolved, which is mandatory for the unambiguous identification of waves and plasma turbulence. The two main methodologies that have been used for wave field characterisation in the frame of Cluster are the $k$-filtering/wave-telescope technique and the phase differencing technique. Important new results obtained through the applications of both techniques to Cluster data have been presented in numerous studies. The purpose of this chapter is not to discuss the scientific results obtained, but to serve as a guide for the interested reader as to what has been learned regarding wave identification methods using multi-spacecraft data and where to find it.

The following is a brief outline of the chapter. In Sections 5.2 and 5.3, respectively, the basic principles of $k$-filtering/wave-telescope and phase differencing techniques are presented. Section 5.4 describes the main problems and limitations encountered by applying the methods to real data. In Section 5.5 the most recent developments of the $k$-filtering and phase differencing techniques are presented and the future of wave field characterisation through multi-spacecraft methods is briefly discussed.

5.2 $k$-filtering — wave-telescope technique

The $k$-filtering technique, also called the wave-telescope technique when applied to magnetic field fluctuations, is described in Chapter 3 of ISSI SR-001 [Pinçon and Motschmann, 1998]. The technique is an adaptation of methods used in geophysics for analysing seismic waves from seismographs distributed over the globe [Capon, 1969]. The three-dimensional generalisation to space plasmas was introduced by Pinçon and Lefèvre [1991]; Pinçon and Lefèvre [1992]. It is a method to characterise stationary fluctuations in space plasmas in terms of the field energy distribution in the frequency and wave vector space.
It is based on simultaneous multi-point measurements of the electromagnetic wave field components, where a filter bank is used to enhance the spatial resolution.

The $k$-filtering/wave-telescope technique was applied for the first time to fluxgate magnetometer data from the Cluster FGM instrument by Glassmeier et al. [2001], to the search-coil magnetometer data from the Cluster STAFF instrument by Sahraoui et al. [2003], and to combined Cluster electric field and magnetometer data from the EFW and STAFF instruments by Tjulin et al. [2005]. The basic principles of the technique are as follows: Let $A(t, r_\alpha)$ be the measured wave field at positions $r_\alpha (\alpha = 1, 2, \ldots)$. Assuming that the measured field is described as a superposition of plane waves, the general expression is given by:

$$A(t, r_\alpha) = \sum_\omega \sum_k A_{\omega,k} \exp(i(k \cdot r_\alpha - \omega t)) + \text{c.c.} \quad (5.1)$$

where $A_{\omega,k}$ is the amplitude of the wave at frequency $\omega$ and wave vector $k$, and c.c. denotes the complex conjugate. The fields are assumed to be stationary in time and homogeneous in space, conditions that in reality are met only approximately. These assumptions may be relaxed by assuming that the fields are translation invariant on spatial scales larger than their wavelengths and stationary on temporal scales greater than the wave period. Moreover, the wave field should not contain waves of a length shorter than the inter-spacecraft separation, otherwise aliasing generates spurious results.

The correlation matrix between the measurements at two positions, $r_\alpha, r_\beta$, obeys the frequency representation

$$M(\omega, r_\alpha, r_\beta) = \langle A(\omega, r_\alpha) A^\dagger(\omega, r_\beta) \rangle \quad (5.2)$$

where $\langle \ldots \rangle$ denotes the time (or ensemble) average. Spatial homogeneity allows linking the correlation matrix $M(\omega, r_{\alpha\beta})$ to the energy distribution matrix $P(\omega, k)$ by

$$M(\omega, r_{\alpha\beta}) = \int P(\omega, k) e^{i k \cdot r_{\alpha\beta}} \, dk \quad (5.3)$$

where $r_{\alpha\beta} = r_\alpha - r_\beta$. Inversion of Eqn. 5.3, i.e., estimating $P(\omega, k)$ from $M(\omega, r_{\alpha\beta})$, is a difficult task since usually the data are spatially undersampled. Managing this crucial problem by constructing a series of non-linear filters is the fundamental goal of $k$-filtering. Each filter is steered to a different $(\omega, k)$ pair and extracts from the data only the energy associated with frequency $\omega$ and wave vector $k$. This $k$-filtering process can also be viewed as a generalised minimum variance analysis [Motschmann et al., 1996].

When constructing the filters, any other useful known information can be exploited. The problem of filter determination can be solved with the help of Lagrange multipliers (see Chapter 3 [Pinçon and Motschmann, 1998] in ISSI SR-001) yielding the following expression for the field energy distribution function $P(\omega, k)$:

$$P(\omega, k) = \Tr \left\{ P(\omega, k) \right\} = \Tr \left\{ [H^T(k) M(\omega)^{-1} H(k)]^{-1} \right\} \quad (5.4)$$

where $H(k)$ is a geometrical matrix depending on the positions of the four satellites. $M(\omega)$ is a matrix containing all correlation matrices $M(\omega, r_{\alpha\beta})$ constructed from the Cluster quartet, as shown in Eqn. 3.7 in Chapter 3 of ISSI SR-001. The fact that the output of
5.3 Phase differencing

Phase differencing is a dispersion-based method relying on phase relations between the data sets obtained from different satellites. It can be applied when simultaneous data from two or more closely spaced satellite are available. A comprehensive description of the method can be found elsewhere [Balikhin et al., 1997a, b; Dudok de Wit et al., 1995]. This method is essentially a generalisation of the phase slowness vector method already mentioned by Born and Wolf [1975] and commonly used in seismology. In what follows a brief description is given.

The basic assumption of the phase differencing method is that the observed wave field can be described as:

\[ A(r, t) = \sum_\omega A_\omega \exp i (k \cdot r - \omega t) + c.c. \]  

(5.5)

where \( A_\omega \) is the Fourier wave amplitude, \( k \) the wave vector (\( k \) and \( \omega \) are related through the wave dispersion relation), and \( c.c. \) denotes the complex conjugate. As for the \( k \)-filtering technique, the wave field should ideally be stationary in time and homogeneous in space, but the same relaxation of those conditions discussed for the \( k \) filtering is assumed, and the same aliasing constraint applies as well. Contrary to the \( k \)-filtering technique, the phase differencing technique is limited to the determination of one wave vector per frequency. This limits its validity to plasma wave field for which most of the wave energy is confined to one particular mode.

Frequency decomposition of the signals \( A(r, t) \) is performed by wavelet decomposition techniques using a Morlet wavelet. This ensures good frequency resolution at the low frequencies that are of interest and a large number of frequency spectra that are used to increase the statistical robustness of the technique [Dudok de Wit et al., 1995]. Hence this method can be used successfully for short periods of data. However, for best results the lowest frequency considered should ensure that there are at least 4–6 wave periods within the data period being analysed.

If the same quantity is measured by two closely spaced satellites \( \alpha \) and \( \beta \), the phase difference at a particular frequency between the two data sets is given by

\[ \Delta \psi(\omega) = \psi_\alpha(\omega) - \psi_\beta(\omega) = (k \cdot r_\alpha - \omega t) - (k \cdot r_\beta - \omega t) = |k||r_{\alpha\beta}| \cos(\theta_{kr}) \]  

(5.6)
where $r_{\alpha\beta}$ is the known separation vector between the two satellites and $\theta_{kr}$ is the angle between the wave vector and satellite separation vector.

The dependence of the phase difference $\Delta \psi$ on $(\omega)$ gives the projection of the wave vector along the separation vector $r_{\alpha\beta}$ as a function of frequency. With the advent of the Cluster mission, data are now available that are measured at four closely separated points in space. This enables the projection of the wave vector to be determined along three independent baselines. It is then possible to reconstruct the original $k$ vector.

5.4 Method successes and limitations

The capabilities of the $k$-filtering/wave-telescope technique for multiple plasma wave mode identification have been successfully demonstrated in many key areas of the terrestrial environment: near Earth solar wind [Glassmeier et al., 2001; Narita et al., 2003], foreshock [Eastwood et al., 2003; Narita et al., 2004], through the bow shock [Narita and Glassmeier, 2005; Narita et al., 2006a], magnetosheath near the magnetopause [Sahraoui et al., 2003, 2004b; Narita and Glassmeier, 2006], high-altitude polar cusp [Grison et al., 2005]. Not only the wave propagation direction and the wave number spectrum could be reconstructed, but also complete wave dispersion analyses or Friedrichs diagram reconstruction were accomplished using the $k$-filtering/wave-telescope technique [Narita et al., 2003; Schäfer et al., 2005]. As shown by Sahraoui et al. [2003], a successful identification of wave-field energy peaks actually due to aliasing can be achieved by comparing the observed wave dispersion relations with the theoretical ones.

The new technique has also been applied with remarkable success to the characterisation of the ULF turbulent magnetic fluctuations observed in the magnetosheath close to the magnetopause [Sahraoui et al., 2004a, 2006] and in the terrestrial foreshock region [Narita et al., 2006b]. The results obtained provide strong arguments for a weak turbulence approach [Belmont et al., 2006]. A crucial point for turbulence theories is to determine the scaling law, which describes how the energy is transferred across spatial scales. Cluster estimates of the wave vector spectra associated with quasi-homogeneous magnetic field turbulence in the magnetosheath suggest that a turbulent cascade is occurring. However, due to aliasing, information about spatial scales smaller than the Cluster inter-spacecraft distance cannot be derived. At the same time, no useful information can be obtained for spatial scales much larger than the mean Cluster inter-spacecraft distance either: it can be shown that in such a case the relative uncertainty for the field energy distribution and the spatial scales are related by $\delta P/P \approx \delta \lambda/\lambda \times d/\lambda$, where $d$ is the mean inter-spacecraft distance. As a consequence, a single tetrahedron mission like Cluster can only cover a limited range of spatial scales at a given time, typically one decade. The investigation of a turbulent cascade, from the injection to the dissipation scale, requires a broader coverage of scales. The only way to cover the various plasma fundamental scales with Cluster is by combining wave vector spectra obtained from $k$-filtering technique using magnetic field data sets performed at different times and for different plasma parameters and solar wind conditions. Any physical conclusion derived from these combined spectra is necessarily linked to very restrictive assumptions.

Initially, the $k$-filtering technique was applied to measurements of the magnetic field by the FGM and STAFF-SC instruments on Cluster. More recently, these data sets have been supplemented with EFW electric field measurements [Tjulin et al., 2005]. The primary
reason behind the inclusion of the electric field data within the $k$-filtering technique is to obtain a better estimate of the wave-field energy distribution. This also enables some comparisons between the wave electric and magnetic field contributions to the wave energy density. These comparisons are useful for basic investigations of the polarisation of the waves. The extended technique has been applied successfully to Cluster data from the magnetosheath and the foreshock. However, its use and the interpretation of the results require an intimate knowledge of the Cluster experiments since there are a large number of problems that require solving before the electric and magnetic field measurements can be combined effectively.

From Cluster data, the phase differencing method was successfully used to identify the mode of plasma waves observed in the magnetosheath [Balikhin et al., 2003a] and in the vicinity of the terrestrial bow shock [Balikhin et al., 2003b]. It has been also applied to Cluster observations of foreshock waves [Hobara et al., 2007a] and solitary structures [Hobara et al., 2007b]. A comparison of both $k$ filtering and phase differencing is presented in Walker et al. [2004]. The results show that both analysis techniques identify the same dominant wave mode in the data and the corresponding $k$ vectors determined are in reasonable agreement. Using a wavelet transform, the dispersion based method can produce good clear results on a shorter period of data than the $k$-filtering technique. The phase differencing method is applied to scalar values measured at different locations such as a component of a vector (e.g., $B_x$ or $E_y$) or oscillations observed in the density. This is in contrast to the $k$-filtering method which requires vectors measured at a minimum of four locations. It was shown that the phase differencing method works best when only one wave mode is present or when one wave mode dominates the wave environment. Multiple modes result in a number of dispersion curves and so the wave-vector directions are currently unresolvable. In contrast, the $k$-filtering technique can resolve multiple waves within the plasma (see Figure 5.1).

5.5 Outlook

The latest development of the phase differencing technique is its application to EFW internal burst data sets. These data sets usually contain the four individual probe potentials sampled up to 9 kHz for a period of 10–11 seconds. Thus, by analysing the phase differences observed in the electric fields on either side of the satellite it is possible to identify waves whose spatial scales are of the order of 100 m or less. This method has been used successfully by Balikhin et al. [2005]. The phase differencing technique can also be used to investigate the polarisation of the waves when it is applied to two perpendicular components of the electric field. This addition to the methodology was used by Walker et al. [2007] to distinguish between circularly polarised whistler mode waves and linearly polarised lower hybrid waves at the front of a quasi-perpendicular bow shock.

A very interesting development of the $k$-filtering/wave-telescope technique can be found by Constantinescu et al. [2006, 2007]. The new tool, called the spherical wave telescope, consists of an extension of the previous technique to spherical waves representation. It provides information about the local curvature of the wave fronts passing through the Cluster satellites. This information is then used to determine the location of the wave source. Another generalisation is presented by Plaschke [2007], who demonstrated that more complex phase front structures such as field-line resonances [Glassmeier et al., 1999] can be incorporated into the wave-telescope technique.
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Figure 5.1: Illustration of the plasma wave mode identification capability of the $k$-filtering technique. In this example, the technique is applied to magnetic field fluctuations in the magnetosheath near the magnetopause, measured by the STAFF instruments on the four Cluster spacecraft. The figure displays the inferred magnetic energy (thin black lines), for a given frequency, in the $(k_x, k_y)$ plane, at two distinct values of $k_z$. The coloured thick lines are the theoretical dispersion relations of the low-frequency modes. The blue line is the Doppler shift $\omega = k \cdot v$. Two main peaks are identified: a mirror mode (top panel) and an Alfvén wave (bottom panel). From Walker et al. [2004].

The recent results from Cluster, and particularly the $k$ spectra determined thanks to the $k$-filtering/wave-telescope technique, demonstrate that future missions will have to be multi-spacecraft in order to produce new insights into the turbulent nature of space plasmas. Spacecraft separations will have to be short enough with respect to the wavelength of the maximum energy fluctuations to remove the aliasing problem. This is already planned for projects like MMS (Magnetospheric Multi-Scale Mission). Information
on larger scales will be provided by projects such as NASA’s five-spacecraft THEMIS mission. In a more distant future, projects of more sophisticated spacecraft clusters, such as the Cross-Scale project proposed in the frame of ESA’s Cosmic Vision, would provide information on the various important scales simultaneously.

Bibliography


